2008 TMSCA STATE MEET NUMBER SENSE

1. $\frac{2}{5} \div \frac{7}{10} \div \frac{3}{14} =$ __________.

Solution : $\left(\frac{2}{5}\right)\left(\frac{10}{7}\right)\left(\frac{14}{3}\right) = \left(\frac{2}{3}\right)\left(\frac{10}{5}\right)\left(\frac{14}{7}\right) = \left(\frac{2}{3}\right)(2)(2) = \frac{8}{3}$

2. The sum of the first 4 composite numbers is __________.

Composite number is a positive integer which has a positive divisor other than one or itself. By definition, every integer greater than one is either a prime number or a composite number. The number one is considered to be neither prime nor composite. For example, the integer 15 is a composite number because it can be factored as 3 x 5.

The following are the first few composite numbers "

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, …

Solution : 4 + 6 + 8 + 9 = 27

3. 42 ÷ 75 = _________ (decimal).

When dividing by 75, multiply by $\frac{4}{3}$ and move the decimal point two places to the left.

Solution : Step #1 : $42\left(\frac{4}{3}\right) = 56$

Step #2 : Move the decimal 2 places to the left. Answer : .56

4. If a bag of candy weighs 12 oz. and sells for $2.40, then a 1 pound bag would cost $_________.

Note : The cost of 1 pound (16 ounces) is the cost of 12 ounces + 4 ounces.

Solution : $\$2.40 + \frac{4}{12}(\$2.40) = \$2.40 + \$.80 = \$3.20$

5. 35 is what percent more than 25? _________%.

35 is 10 more than 25. $\frac{10}{25} = \left(\frac{10}{25}\right)(100)\,\% = 40\%$

6. If GCD(40, k) = 8 and LCM(40, k) = 120, then k is _________.

Note : The product of the GCD and LCM of a and b is equal to ab.

Solution : 8(120) = 40k

$$k = \frac{8(120)}{40} = 8(3) = 24$$

7. $(8)^{-1}$ x $(8)^{2} \div (8)^{-3}$ = _________.

Solution : $\frac{(8)^{-1}(8)^{2}}{(8)^{-3}} = (8)^{(-1)+(2)-(-3)} = (8)^{4} = 4096$

8. What number times six and added to four, gives the same result?

Solution : $6x = x + 4$

$5x = 4$; $x = \frac{4}{5}$ or .8

9. $|(2-5)+|-3|-7| = |-3+3-7| = |-7| = 7$

10. $24^2 + 8^2$ = ________.

Note : $(3x)^2 + (x)^2 = 10x^2$

Solution : $10(8)^2 = 10(64) = 640$

11. 235 base 7 is equivalent to ________ base 10.

$7(7(2) + 3) + 5 = 7(17) + 5 = 119 + 5 = 124$

12. $1 + 4 + 7 + 10 + 13 + \ldots + 28$ = _________.

This is an arithmetic series with difference of 3.

Sum of an arithmetic series $= \frac{n}{2}(a + t_n)$

Solution : Step #1 : Determine the number of terms. $1 + 3k = 28$; $3k = 27$; $k = 9$

Number of terms $= 9 + 1 = 10$

Step #2 : $\frac{10}{2}(1+28) = 5(29) = 145$

13. Let $x = 2$, $y = 3x$, and $z = y - x$. Find xyz. ________.

$z = y - x = 3x - x = 2x$

$xyz = x(3x)2x) = 6x^3 = 6(2)^3 = 6(8) = 48$

14. If the Universal set U = {u, n, i, v, e, r, s, a, l} and set A = {l. i. v. e. r}, then the complement of set A contains how many distinct elements? _________.

The complement of set A is the set of elements in the Universal set that are not in A.
the complement of set A = {u, n, s, a} which has 4 distinct elements.

15. If $3x - 5 = -7$, then $6x =$ ________.

Solution : $3x - 5 = -7$; $3x = -2$; $2(3x) = 2(-2)$; $6x = -4$

16. $1.08333\ldots + 1.58333\ldots =$ _________.

Memorize : $\frac{1}{12} = 8\frac{1}{3}\% = .08333\ldots$; $\frac{7}{12} = 58\frac{1}{3}\% = .58333\ldots$

Solution : $1\frac{1}{12} + 1\frac{7}{12} = 2\frac{8}{12} = 2\frac{2}{3}$

17. $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 =$ _________.

Note : $a^2 + b^2 + (a + b)^2 + (a + 2b)^2 + \ldots + c^2 + d^2 = d(c + d) - a(b - a)$

Solution : $21(21 + 13) - 1(1 - 1) = 21(34) = 714$

Example A : $1^2 + 3^2 + 4^2 + 7^2 + 11^2 + 18^2 =$ _________.

Solution : $18(18 + 11) - 1(3 - 1) = 18(29) - 2 = 522 - 2 = 520$

Example B : $2^2 + 5^2 + 7^2 + 12^2 + 19^2 =$ _________.

Solution : $19(19 + 12) - 2(5 - 2) = 19(31) - 2(3) = 589 - 6 = 583$

Example C : $6^2 + 3^2 + 9^2 + 12^2 + 21^2 =$ _________.

Solution : $21(21 + 12) - 6(3 - 6) = 21(33) + 18 = 693 + 18 = 711$

18. A square has a diagonal of $6\sqrt{2}$ inches. The area of the square is ________ sq. inches.

Note : The area of a square is equal to one half the square of the diagonal.

Solution : $\frac{1}{2}\left(6\sqrt{2}\right)^2 = \frac{1}{2}(72) = 36$

19. The sum of the product of the roots taken two at a time of $6x^3 - 13x^2 = 2 - x$ is _________.

Note : If $ax^3 + bx^2 + cx + d = 0$, then the sum of the product of the roots taken two at a time is $\frac{c}{a}$.

Solution : $6x^3 - 13x^2 + x - 2 = 0$

$$\frac{c}{a} = \frac{1}{6}$$

*20. $\sqrt[3]{4910}$ x $\sqrt{255}$ x 15 = _________.

Note : $\sqrt[3]{4913}$ = 17 ; $\sqrt{256}$ = 16

Solution : $\sqrt[3]{4910}$ x $\sqrt{255}$ x 15 is approximately greater than 17 x 16 x 15 =

272 x 15 = 272(10 + 5) = 2720 + 1360 = 4080

Answer should be greater than 3840. Note : Range : 3868 - 4274

21. The smaller leg of a right triangle with integral sides is 7". The triangle's perimeter is ________".

If the smaller leg is a negative number greater than or equal to 3, to the following :

Step #1 : Square the number.

$7^2 = 49$

Step #2 : Find two numbers whose sum is equal to the result of Step #1.

49 = 24 + 25

The integral sides of the right triangle are 7, 24 and 25.

Perimeter = 7 + 24 + 25 = 56

Example A : The smaller leg of a right triangle with integral sides is 5". The triangle's Perimeter is ________".

Step #1 : $5^2 = 25$

Step #2 : 25 = 12 + 13

perimeter = 5 + 12 + 13 = 30

Example B : The smaller leg of a right triangle with integral sides is 11". The triangle's perimeter is ________".

Step #1 : $11^2 = 121$

Step #2 : 121 = 60 + 61

Perimeter = 11 + 60 + 61 = 132

Example C : The smaller leg of a right triangle with integral sides is 13". The triangle's perimeter is ________".

Step #1 : $13^2 = 169$

Step #2 : 169 = 84 + 85

Perimeter : 13 + 84 + 85 = 182

Notice : Given side, a, as the shorter leg of a right triangle, its perimeter is equal to a(a + 1).

22. The number of distinct diagonals of a regular hexagon is _________.

Note : A regular n-gon has $\frac{n(n-3)}{2}$ distinct diagonals.

Solution : $\frac{6(6-3)}{2} = \frac{6(3)}{2} = \frac{18}{2} = 9$

23. 90 x 5! + 150 x 4! = _________.

4!(90 x 5 + 150) = 4!(450 + 150) = 4!(600) = 24(600) = 14400

24. $\frac{9}{11} - \frac{33}{47}$ = _________.

Note : $\frac{a}{b} - \frac{4a-3}{4b+3} = \frac{4ab+3a-4ab+3b}{b(4b+3)} = \frac{3(a+b)}{b(4b+3)}$

Solution : Step #1 : Find the product of 3 and the sum of the numerator and denominator of the fraction on the left. The result is the numerator of the answer.

3(9 + 11) = 3(20) = 60

Step #2 : Find the product of the denominator. The result will be the denominator of the answer.

11(47) = 517

25. $35\frac{5}{7}\%$ = _________ (proper fraction).

Note : $\frac{1}{14} = 7\frac{1}{7}\%$; $5\left(\frac{1}{14}\right) = 5\left(7\frac{1}{7}\%\right)$; $\frac{5}{14} = 35\frac{5}{7}\%$

26. The point (- 1, 2) is reflected across the x-axis to point (h, k). Find k. ________.

Note : If the point (x, y) is reflected across the x-axis, the result will be the point (x, - y).

Solution : - 2

27. If $x^2 + 6 > 11$, then $2x^2 - 4 >$ _________.

Solution : $x^2 + 6 > 11$

$x^2 > 11 - 6$

$x^2 > 5$

$2x^2 > 2(5)$

$2x^2 - 4 > 10 - 4$

$2x^2 - 4 > 6$

Answer : 6

28. If $\sqrt{12} + \sqrt{27} = \sqrt{x}$, then x = _________.

Solution : $\sqrt{4(3)} + \sqrt{9(3)} = \sqrt{x}$; $2\sqrt{3} + 3\sqrt{3} = \sqrt{x}$; $5\sqrt{3} = \sqrt{x}$;

$\sqrt{25(3)} = \sqrt{x}$; $\sqrt{75} = \sqrt{x}$; x = 75

29. $\ln e^5$ = _________.

Note : $\ln e^p = p\ln e = p(1) = p$ ($\ln e = 1$).

Solution : $\ln e^5 = 5\ln e = 5(1) = 5$

30. The probability of winning is $\frac{7}{11}$. The odds of losing is _________.

Probability = $\frac{Favorable}{TotalOutcomes}$

Unfavorable = Total Outcomes - Favorable

Solution : The odds of losing = $\frac{11-7}{7} = \frac{4}{7}$

31. Let $x^2 = \sqrt{2x^3 + 2x^3 + 2x^3 + 2x^3}$, where x > 0. Find x. _________.

Solution : $\left(x^2\right)^2 = \left(\sqrt{2x^3 + 2x^3 + 2x^3 + 2x^3}\right)^2$; $x^4 = 8x^3$; x = 8

32. The sum of the coefficients of $(2x + 2y)^3$ is _________.

Note : The sum of the coefficients of $(ax + by)^n$ is $(a + b)^n$.

Solution : $(2 + 2)^3 = 4^3 = 64$

33. If (2 + 3i) ÷ i = a + bi, then a = ________.

If you are looking for a, simply divide the i term by i.

$\frac{3i}{i} = 3$

*34. Approximate : 13 x 17 x 21 x 25 = ________.

Use the shortcut for multiplying two numbers whose units digits add up to 10 and whose other digits are the same to multiply 13 x 17.

13 x 17 = 221

Approximation : Slightly more than : 221 x 21 x 15 = 220 x 20 x 15 = 4400 x 25 = 110.000

Range of answer : 110,224 - 121,826

35. The larger root of $42x^2 + 59x - 33 = 0$ is _________.

Solution : $(6x + 11)(7x - 3) = 0$

$x = -\frac{11}{6}$ or $\frac{3}{7}$; Answer : $\frac{3}{7}$

36. $\det\begin{vmatrix} -3 & -5 \\ 2 & 7 \end{vmatrix}$ = ________.

Note : $\det\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Solution : (- 3)(7) - (2)(- 5) = (- 21) + 10 = - 11

37. If the initial point of a vector is (- 2, 14) and the terminal point of (- 5, 10), then $\|v\|$ = _________.

Solution : $\|v\| = \sqrt{(-2--5)^2+(14-10)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$

38. 111111 = 12345 x 9 + _________.

Note : 12 x 9 + 3 = 111

123 x 9 + 4 = 1,111

1234 x 9 + 5 = 11.111

12345 x 9 + 6 = 111,111

123456 x 9 + 7 = 1,111,111

Solution : Answer : 6

39. A square based pyramid has a base side length 3' and a height 12'. Its volume is ________ cu. ft.

Volume = $\frac{1}{3}$Bh, where B is the area of the base and h is the height

Solution : $\frac{1}{3}(3)^2(12) = \frac{1}{3}(108) = 36$

40. $55^2 - 56^2 + 57^2 - 58^2 =$ _________.

Type #1 : If the terms being squared differ by d and are decreasing the answer is equal to - d(sum of the terms).

Solution : $-1(55 + 56 + 57 + 58) = -226$

Type #2 : IF the terms being squared differ by d and are increasing the answer is equal to d(sum of the terms).

Example A : $22^2 - 19^2 + 16^2 - 13^2 =$ _________.

Solution : $3(22 + 19 + 16 + 13) = 3(70) = 210$

41. The remainder of $f(x) = 3x^2 + 4x + k$ divided by $x + 2$ is 15. Find k.

The remainder of $f(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots$ is divided by $x - k$ is equal to f(k).

Solution : $f(-2) = 3(-2)^2 + 4(-2) + k = 15$

$12 - 8 + k = 15$; $4 + k = 15$; $k = 11$

42. The greatest integer function $f(x) = [x-5]$ has a value of ________ for f(3.14).

Solution : $[3.14-5] = [-1.86]$; The greatest integer less than or equalo to – 1.86 is – 2.

*43. $3.14\pi \text{ x } 2.72e \text{ x } 1.62\phi =$ ________.

Note : π is approximately equal to 3.14, e is approximately equal to 2.72 and ϕ is the Golden Mean ($\frac{1+\sqrt{5}}{2}$) and is approximately 1.62.

Memorize : πe is approximately equal to 8.5 ‘ $\pi e \phi$ is approximately equal to 13.8

Solution : $3.14\pi \text{ x } 2.72e \text{ x } 1.62\phi$ is approximately equal to $(3.14 \text{ x } 2.72 \text{ x } 1.62)^2 = [8.5(1.6)]^2 =$

$[17(.8)]^2 = 13.6^2$ which is slightly less than $14^2 = 196$

Range of answer is 182 - 200

44. Change .42 base 6 to a base 10 fraction. ________.

$.42 \text{ base } 6 = \frac{4}{6} + \frac{2}{36} = \frac{24}{36} + \frac{2}{36} = \frac{26}{36} = \frac{13}{18}$

45. The largest value of x in the domain of f(x) so that f(x) = $\sqrt{5-4x}$ has a real valued range is ________.

Solution : 5 - 4x = 0 ; 5 = 4x ; x = $\frac{5}{4}$

46. Find x, 0 ≤ x < 6, if $\frac{(5!)(2!)}{(4!)}$ ≅ x(mod 6). ________.

Solution : x is equal to the remainder when $\frac{(5!)(2!)}{(4)!}$ is divided by 6.

$\frac{(5!)(2!)}{(4!)}$ = 10 ; The remainder when 10 is divided by base 6 is 4.

Answer : 4

47. $\lim_{x\to 0} \frac{\sin 2x}{x}$ = ________.

Step #1 : Let x = 0 in $\frac{\sin 2x}{x}$. The result is $\frac{0}{0}$.

Step #2 : Since the result in Step #1 was an indiscriminant, apply L'Hospitals Rule.

Step #2 : $\lim_{x\to 0} \frac{2\cos 2x}{1}$ = 2cos[2(0)] = 2cos0 = 2(1) = 2

48. The horizontal asymptote of y = $\frac{2x+1}{x-3}$ is ________.

Solution : $\lim_{x\to\infty} \frac{2x+1}{x-3}$ = $\frac{2}{1}$ = 2 (Note : When evaluating the limit when x approaches infinity, the limit is equal to the coefficient of the leading term in the numerator over the coefficient of the leading term in the denominator, if the degree in the numerator and denominator are equal, and if the terms in the numerator and denominator are both in standard form.

49. If f(x) = $4x^3$ - 3x + 2, then f''(- 1) = _________.

Solution : f'(x) = $12x^2$ - 3

f''(x) = 24x

F''(- 1) = 24(- 1) = - 24

50. $\frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21}$ = _________.

The triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, …

The sum of x reciprocal of consecutive triangular numbers starting with the nth triangular number is $\frac{2x}{n(n+x)}$.

Solution : $\frac{2(4)}{3(3+4)} = \frac{8}{21}$