

2002 - 2004 UIL NUMBER SENSE SHORTCUTS

SAMPLE PAGES

1. $321 - 123 = \underline{\hspace{2cm}}$.

Step #1 : Find the difference between the units digit and the hundreds digit.

$$3 - 1 = 2$$

Step #2 : Multiply the result from Step #1 by 99. Note : It is easier if you can picture multiplying the result from Step #1 by $100 - 1$.

$$2(100 - 1) = 200 - 2 = 198$$

2. $2 + 4 + 6 + 8 + \dots + 28 + 30 = \underline{\hspace{2cm}}$.

Step #1 : Divide the last digit by 2.

$$30 \div 2 = 15$$

Step #2 : Find the product of the result of Step #1 by that same result increased by 1.

$$15(15 + 1) = 15(16) = 240$$

3. $421 \div 9 = \underline{\hspace{2cm}}$ (mixed number)

Step #1 : Find the sum of the digits of the given number. This will be the numerator of a fraction whose denominator is 9. If the result is a proper fraction write this as part of the answer. If the result is an improper fraction, convert it into a mixed number ; write down the fractional part, then carry the whole number part to Step #2.

$$\frac{4 + 2 + 1}{9} = \frac{7}{9}$$

Step #2 : Find the sum of the hundreds digit and the tens digit (plus any carryover from Step #1). This is the units digit of the answer.

$$(4 + 2) + 0 = 6$$

Step #3 : The hundreds digit of the number being divided by 9 (plus any carryover from Step #2) will be the tens digit of the answer.

$$4 + 0 = 4$$

$$\text{Answer : } 46 \frac{7}{9}$$

13. $\frac{5}{6} - \frac{5}{12} - \frac{5}{24} = \underline{\hspace{2cm}}$.

Note : $\frac{a}{b} - \frac{a}{2b} - \frac{a}{4b} = \frac{a}{4b}$

Solution : The answer is the last fraction. $\frac{5}{24}$

14. $\frac{5}{7} + \frac{7}{5} = \underline{\hspace{2cm}}$ (mixed number).

Step #1 : Find the square of the difference of the numerator and the denominator.

$$(7 - 5)^2 = 2^2 = 4$$

Step #2 : Write the result from Step #1 over the product of the two denominators.

$$\frac{4}{7 \times 5} = \frac{4}{35}$$

If the result of Step #2 is improper, convert into a mixed number and write down the fractional part and carryover the whole number part to Step #3.

Step #3 : Write down 2 plus any carryover from Step #2.

$$2 \quad ; \quad \text{Answer : } 2 \frac{4}{35}$$

Example : $\frac{12}{13} + \frac{13}{12} = \underline{\hspace{2cm}}$ (mixed number).

Step #1 : Find the difference of the numerator and the denominator.

$$13 - 12 = 1$$

Step #2 : Square the result of Step #1 and write it over the product of the denominator of the fractions.

$$\frac{1^2}{13(12)} = \frac{1}{156}$$

60. $\frac{11}{30} + \frac{11}{42} + \frac{11}{56} = \underline{\hspace{2cm}}$.

Note : $\frac{11}{30} + \frac{11}{42} + \frac{11}{56} = \frac{11}{5(6)} + \frac{11}{6(7)} + \frac{11}{7(8)}$

$$\frac{a}{b(b+1)} + \frac{a}{(b+1)(b+2)} + \frac{a}{(b+2)(b+3)} = \frac{3a}{b(b+3)}$$

Solution : $\frac{3(11)}{5(5+3)} = \frac{33}{5(8)} = \frac{33}{40}$

Example : $\frac{5}{12} + \frac{5}{20} + \frac{5}{30} = \underline{\hspace{2cm}}$.

Solution : $\frac{3(5)}{3(3+3)} = \frac{15}{3(6)} = \frac{15}{18} = \frac{5}{6}$

61. $1111 \times 123 = \underline{\hspace{2cm}}$.

Step #1 : The units digit of the answer is equal to the units digit of the number being multiplied by 1111.

Units digit is 3.

Step #2 : The tens digit of the answer is equal to the sum of the units digit and the tens digit of the number being multiplied by 1111. If you get a 2-digit number, write down the units digit and carryover the tens digit to Step #3.

$2 + 3 = 5$. The tens digit is 5.

Step #3 : The hundreds digit of the answer is equal to the sum of the three digits of the number being multiplied by 1111. If the result is a 2-digit number, write down the units digit and carryover the tens digit to Step #4.

$1 + 2 + 3 = 6$. The hundreds digit is 6.

Step #4 : The thousands digit of the answer is equal to the sum of the three digits of the number being multiplied by 1111. If the result is a 2-digit number, write down the units digit and carryover the tens digit to Step #5.

$1 + 2 + 3 = 6$. The thousands digit is 6.

Step #5 : The thousands digit of the answer is equal the sum of the hundreds digit and the tens digit of the number being multiplied by 1111. If the result is a 2-digit number, write down the units digit and carryover the tens digit to Step #6.

$1 + 2 = 3$. The ten thousands digit is 3.

Step #6 : The remainder of the answer is equal to the hundreds digit of the number being multiplied by 1111 (plus any carryover).

The last digit of the answer is 1.

62. If the area of an equilateral triangle is $9\sqrt{3}$ sq. cm., then its side length is _____ cm.

$$\text{Rule : Length of side} = \sqrt{\frac{4 \times \text{Area}}{\sqrt{3}}}$$

$$\text{Solution : } \sqrt{\frac{4 \times 9\sqrt{3}}{\sqrt{3}}} = \sqrt{4(9)} = \sqrt{36} = 6$$

63. $1 + 2 + 2^2 + 2^3 + \dots + 2^7 =$ _____.

$$1 + a + a^2 + a^3 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

$$\text{Solution : } \frac{2^{1+1} - 1}{2 - 1} = \frac{2^8 - 1}{1} = 255$$

Example : $1 + 3 + 3^2 + 3^3 + \dots + 3^5 =$ _____.

$$\text{Solution : } \frac{3^{5+1} - 1}{3 - 1} = \frac{3^6 - 1}{2} = \frac{728}{2} = 364$$

64. A tin can a inches high holds b ounces. A similar tin can c inches high holds _____ oz.

$$\text{Rule : } \left(\frac{c}{a}\right)^3 \times b$$

Example : A tin can 4 inches high holds 6 ounces. A similar tin can 8 inches high holds _____ oz.

$$\text{Solution : } \left(\frac{8}{4}\right)^3 \times 6 = 8 \times 6 = 48$$

65. The diagonal of a square is \sqrt{a} units. Find the length of the side of the square.

Rule : Side = $\sqrt{\frac{a}{2}}$

Example : The diagonal of a square is $\sqrt{18}$ units. Find the length of the side of the square.

Solution : $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$