

MIDDLE SCHOOL MATHEMATICS MAGIC

BOOK I

BASIC MATHEMATICS

1. If 120% of A equals $\frac{2}{5}$ of B and 25% of B equals C, then A is what part of C?

(A) $\frac{4}{3}$ (B) $\frac{6}{5}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{8}{5}$

$$120\% = \frac{6}{5} \text{ and } 25\% = \frac{1}{4}$$

$$\text{Let } \frac{1}{4}B = C \text{ and } \frac{6}{5}A = \frac{2}{5}B ; A = \frac{5}{6}\left(\frac{2}{5}\right)B = \frac{1}{3}B$$

$$\frac{A}{C} = \frac{\frac{1}{3}B}{\frac{1}{4}B} = \frac{\frac{1}{3}}{\frac{1}{4}} = \frac{1}{3} \div \frac{1}{4} = \frac{1}{3}\left(\frac{4}{1}\right) = \frac{4}{3}$$

2. Sweet Sher has a box of 36 candy canes. Her mother takes one third of the box and puts it away for another day. She divides the remaining candy canes into 3 equal parts for their sister, her brother, and herself. How many candy canes did Sher keep for herself?

(A) 24 (B) 18 (C) 16 (D) 12 (E) 8

Sher's mother kept one third of 36 or 12 candy canes. Each of the children then got one third of the 24 remaining candy canes or 8 each.

3. Tommy buys 4 calculators for his math team at a price of \$83.49 each. Each calculator requires 3 batteries which sell for \$1.89 for a package of 2 batteries. The tax rate is $7\frac{1}{4}\%$. What will the total cost be including tax? (to the nearest cent)

(A) \$364.25 (B) \$370/33 (C) \$345.36 (D) \$382.50 (E) \$366.28

Instead of finding the sales tax and adding it to the subtotal, I would suggest that you multiply the subtotal by the sales tax plus 100%, which in this case is

$$107 \frac{1}{4}\% = 1.0725.$$

$1.0725[4(83.49) + 6(1.89)] = 370.33$ (Note : Since each calculator requires 3 batteries and you were buying 4 calculators, you needed to purchase 12 batteries or 6 packages of 2).

4. $55 \text{ mph} = \underline{\hspace{2cm}} \text{ ft/sec.}$

- (A) $48 \frac{2}{5}$ (B) $37 \frac{1}{2}$ (C) 83 (D) 70 (E) $80 \frac{2}{3}$

$$55 \frac{\text{miles}}{\text{hour}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} = 55 \left(\frac{22}{15} \right) = \frac{242}{3} = 80 \frac{2}{3}$$

Keep in mind that when converting from mph to ft/sec you should multiply the miles per hour by $\frac{22}{15}$. If converting from ft/sec to mph you multiply the ft/sec by $\frac{15}{22}$.

5. Robin went to the mall for a special sale. She bought 3 sweaters at \$19.95 each, 2 pairs of pants at \$17.50 each, and a music CD for \$8.99. The total cost including tax was \$110.85. What was the tax rate?

- (A) 4.2% (B) $6 \frac{3}{4}\%$ (C) 10.7% (D) $7 \frac{1}{4}\%$ (E) 8.125%

Let x = the tax rate plus 100% (in decimal form)

$$x[3(19.95) + 2(17.50) + 8.99] = 110.85$$

$$x(103.84) = 110.85$$

$$x = \frac{110.85}{103.84} = 1.0675 ; \text{ Thus the tax rate is } .0675 = 6.75\% = 6 \frac{3}{4}\%$$

6. The arithmetic mean of all of the positive integral divisors greater than one of the number 27 is :

- (A) 9 (B) 10 (C) 13 (D) 22 (E) 27

The arithmetic mean of a set of numbers is also known as the mean or average of the set of numbers. The positive integral divisors of a number is the set of positive integers that divide a number without a remainder.

The positive integral divisors greater than 1 of 27 are 3, 9 and 27. The arithmetic mean of these numbers is $\frac{3+9+27}{3} = \frac{39}{3} = 13$

7. The new millennium began on Monday, January 1, 2001. Pearl Harbor Day's 60th anniversary was December 7, 2001. What day of the week did the 60th anniversary fall on?

(A) Sunday (B) Monday (C) Tuesday (D) Thursday (E) Friday

The year 2001 was not a leap year so it had 365 days. If you consider January 1 as day 1 of the year, then December 31 was day 365. Since December 7 is 24 days before December 31, then December 7 would be considered day 341. To find day of the week, Keep in mind that when going from January 1 (day 1) to December 7 (day 341) there are 340 days (48 weeks and 4 days). If January 1 was a Monday, 48 weeks later it would still be a Monday. Since you need to find the day of the week for December 7, you must go four more days which makes it a Friday.

- 8 CD Shoppe is having a sale on music CD's that regularly cost \$12.50 each. If you buy three you get the fourth one free. Also, as a bonus, you get the fifth one at half price. If you buy five CD's what is the average cost for each one? (ignore tax)

(A) \$11.25 (B) \$8.75 (C) \$9.38 (D) \$10.50 (E) \$7.94

$$\text{Average cost} = \frac{\text{total cost}}{\text{number bought}} = \frac{3.5(12.50)}{5} = 8.75$$

9. The sum of three consecutive multiples of three equals 144. The smallest of the multiples is :

(A) 39 (B) 42 (C) 45 (D) 48 (E) 51

Let x = the smallest integer, $x + 3$ be the next largest integer and $x + 6$ be the largest integer.

$$x + (x + 3) + (x + 6) = 144 ; 3x + 9 = 144 ; 3x = 135 ; x = 45$$

10. The first day of the week of the 2005 new year was on a Saturday. What day of the week did the 2004 year begin?

(A) Sunday (B) Monday (C) Tuesday (D) Thursday (E) Friday

Since 2004 was a leap year it had 366 days. If January 1, 2004 was day 1, then January 1, 2005 was day 367. The difference in the number of days of these two dates is 366 or 52 weeks 2 days. If January 1, 2005 was a Saturday, 52 weeks earlier it was also a Saturday. Two days before that would have been a Thursday.

11. In the decimal number $33xy2y$, the letters x and y represent digits. If the number is divisible by 275, then x is :

(A) 0 (B) 4 (C) 5 (D) 7 (F) 8

Assume that x , and y are all 0. $330020 \div 275 = 1200.0727$, thus the question can be thought of as $xy2y \div 275 = 0$. Keep in mind that since y is the ones place, y must equal 0 or 5. Thus the last three digits must be 020 or 525. Try and see what number $x020$ or $x525$ is divisible by 275.

$$9020 \div 275 = 32.8$$

$$9525 \div 275 = 34.6364$$

$$8020 \div 275 = 29.1636$$

$$8525 \div 275 = 31 \quad (\text{Thus the number is } 338525 \text{ is the answer, since } 338525 \div 275 = 1231).$$

12. Spend Les went to the $33\frac{1}{3}\%$ Off Super CD Sales. Les only has \$85.00 to spend. The regular price for each CD is \$17.50 and the sales tax rate is 8.25%. What is the greatest number of CD's Les can buy?

(A) 14 (B) 13 (C) 7 (D) 6 (E) 4

A $33\frac{1}{3}\%$ discount means she will only pay $66\frac{2}{3}\%$ or $\frac{2}{3}$ of the cost.

To add 8.25% sales tax, multiply the cost by 1.0825

Let x = the number of CD's she will buy.

$$\frac{2}{3}(x)(17.50)(1.0825) = 85 \quad ; \quad x = \frac{3}{2} \left(\frac{85}{17.50(1.0825)} \right) = 6.7305 \quad (\text{This means}$$

she can afford to buy 6 CD's)

- 13, Betty Wont lost 48% of the first 25 races. How many of the next 12 races must Betty win in order to have won at least 50% of the total races?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

If Betty lost 48% of the first 25 races, she lost $.48(25) = 12$ races, which implies

she won 13 races. If she races 12 more times this will mean she will have raced 37 races. To win at least 50%, she must win at least 19 races. Since she has already won 13 races, she needs to win 6 of the next 12 races.

14. Sher Ing baked some cookies. Sher gave $\frac{1}{5}$ of them to her sister. She put 30% of what was left in a box for her grandmother. She had 14 left. How many cookies did Sher bake?

- (A) 20 (B) 22 (C) 25 (D) 28 (E) 30

Let x = the number of cookies

$$\frac{1}{5}x + \frac{3}{10}\left(\frac{4}{5}x\right) + 14 = x ; \quad \frac{1}{5}x + \frac{12}{50}x + 14 = x ;$$

$$x - \left(\frac{1}{5}x + \frac{12}{50}x\right) = 14 ; \quad x - \left(\frac{10}{50}x + \frac{12}{50}x\right) = 14 ; \quad \frac{28}{50}x = 14 ;$$

$$x = \frac{50}{28}(14) = 25$$

15. Doug Down rented a ditch digger for 8 hours. The rental fees were \$12.00 per hour for the first 5 hours and \$6.00 per hour for the last 3 hours. Doug received a \$4.00 rebate for returning the digger before closing time. The tax rate is $8\frac{1}{2}\%$. What was Doug's total cost?

- (A) \$67.27 (B) \$71.61 (C) \$80.29 (D) \$84.63 (E) \$88.97

To find the total cost including tax multiply cost of the digger by $100\% + 8\frac{1}{2}\%$.

$$1.085[(12)(5) + (6)(3) - 4] = 80.29$$

16. Rene Thorn went to the Oak Tree Nursery to buy some fruit trees. The trees were being sold for \$24.95. She bought an apple tree, two peach trees, and a pear tree. Since Rose belonged to the garden club she received a 15% discount. she gave the teller a \$100 bill and received \$9.02 change. What was the tax rate? (nearest quarter per cent)

ALGEBRA I

1. Set $A = \{2, 3, 5, 7, 11, \dots\}$. Which of the following is not an element of set A?

(A) 133 (B) 113 (C) 103 (D) 73 (E) 53

The elements of Set A are prime numbers. The only number given that is not a prime number is 133. Note : $133 = 7(19)$

2. Find the value of k when $(-4, 1)$ lies on the line $kx + 3y = 4$.

(A) -1.25 (B) .75 (C) -1.75 (D) -.25 (E) 1.75

$$k(-4) + 3(1) = 4 \quad ; \quad -4k = 4 - 3 \quad ; \quad k = -\frac{1}{4} = -.25$$

3. A 20-pound bag of Cheep Brand cement mix contains 25% cement and 75% sand. How much pure cement must be added to produce a cement mix that is 40% cement?

(A) 5 lbs. (B) 15 lbs. (C) 20 lbs. (D) 50 lbs. (E) 55 lbs.

$$25\%(20) + 100\%(x) = 40\%(x + 20)$$

$$25(20) + 100x = 40x + 40(20)$$

$$500 + 100x = 40x + 800$$

$$100x - 40x = 800 - 500$$

$$60x = 300$$

$$x = 5$$

4. Let M and N be the roots of $3x^2 - 5x + 2 = 0$. Find M^2N^2 .

(A) $2\frac{7}{9}$ (B) $\frac{4}{9}$ (C) $\frac{4}{5}$ (D) $\frac{5}{9}$ (E) $2\frac{1}{4}$

Note : $M^2N^2 = (MN)^2$, which is asking for the square of the product of the roots. If $Ax^2 + Bx + C = 0$, the product of the roots is $\frac{C}{A}$. The

product of the roots of $3x^2 - 5x + 2 = 0$ is $\frac{2}{3}$ and the square of the

product of the roots is $\frac{4}{9}$.

5. $P = \{a, e, i, o, n\}$, $Q = \{p, o, e, m\}$ and $R = \{r, h, y, m, e\}$, then $(P \cup Q) \cap (Q \cup R)$ equals :

- (A) $\{e\}$ (B) $\{m, o, e\}$ (C) $\{r, h, y, m, e\}$
 (D) $\{p, o, e, m\}$ (E) $\{a, e, i, o, u\}$

$P \cup Q$ is the union of the two sets, P and Q. Find the set that contains all of the elements of both sets : $\{a, e, i, o, n, p, m\}$. $Q \cup R$ is the union of the two sets Q and R. The union is $\{p, o, e, m, r, h, y\}$.

$(P \cup Q) \cap (Q \cup R)$ can be found by finding the intersection of the two previous answers. To find the intersection, find the set whose elements are found in both sets.

$$\{a, e, i, o, n, p, m\} \cap \{p, o, e, m, r, h, y\} = \{p, o, e, m\}$$

6. If $5(3 + 4) = 5(7)$ and $5(7) = 35$, then $5(3 + 4) = 35$. Which of the following properties does this example illustrate?

- (A) commutative (B) associative (C) symmetric
 (D) transitive (E) distributive

commutative property : $a + b = b + a$, $ab = ba$

associative property : $(a + b) + c = a + (b + c)$

symmetric property : If $a = b$, then $b = a$.

transitive property : If $a = b$ and $b = c$, then $a = c$

distributive property : $a(b + c) = ab + ac$

The property illustrated by the example is the transitive property.

7. Simplify : $[b + \frac{b}{a}] \div [b - \frac{b}{a}]$

- (A) $a - b$ (B) $\frac{a+1}{a-1}$ (C) $a + b$ (D) $\frac{a+b}{b-a}$ (E) $\frac{1+b}{1-1}$

I suggest that you rewrite the problem as a fraction :

$\frac{b + \frac{b}{a}}{b - \frac{b}{a}}$ then multiply the numerator and the denominator by a .

$$\left(\frac{b + \frac{b}{a}}{b - \frac{b}{a}}\right)\left(\frac{a}{a}\right) = \frac{ab + b}{ab - b} = \frac{b(a + 1)}{b(a - 1)} = \frac{a + 1}{a - 1}$$

8. The sum of three positive consecutive integers is 165. The largest of the three numbers is _____.
- (A) 60 (B) 55 (C) 53 (D) 52 (E) 56

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Let x , $x + 1$ and $x + 2$ be the three positive consecutive integers.

$$x + (x + 1) + (x + 2) = 165$$

$$3x + 3 = 165$$

$$3x = 162$$

$x = 54$ Since x is the smallest integer, then $x + 2$ or 56 is the largest of the three numbers.

A quicker way you may have done this problem, is to find the average of the numbers (Note : This would have given you the integer in the middle). Then increase the result by 1.

$$\frac{165}{3} + 1 = 55 + 1 = 56$$

9. Line l going through points $(1, 2)$ and $(3, a)$ is parallel to $2x - 3y = 4$. What is the value of a ?
- (A) -1 (B) $\frac{2}{3}$ (C) $3\frac{1}{3}$ (D) 5 (E) $-\frac{4}{3}$

Keep in mind that if two lines are parallel they have the same slope. Given $ax + by = c$, $by = -ax + c$ and $y = -\frac{a}{b}x + \frac{c}{b}$, where $-\frac{a}{b}$ is the slope and $\frac{c}{b}$ is the y-intercept of the line. The slope of $2x - 3y = 4$ is

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$\frac{2}{3}$. Use the slope formula ($m = \frac{y_2 - y_1}{x_2 - x_1}$) to solve for a.

$$\frac{2}{3} = \frac{a-2}{3-1} ; 2(3 - 1) = 3(a - 2) ; 4 = 3a - 6 ; 3a = 10 ;$$

$$a = \frac{10}{3} = 3 \frac{1}{3}$$

10. The discriminant of $5x^2 - 4x - 1 = 0$ is _____.

- (A) 36 (B) 4 (C) 6 (D) 24 (E) -4

If $ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$. The discriminant of $5x^2 - 4x - 1 = 0$ is $(-4)^2 - 4(5)(-1) = 16 + 20 = 36$

11. An equation of a line with a y-intercept of 5 and slope of 2 is _____.

- (A) $y - 2x = -5$ (B) $2x - 5 = y$ (C) $2x - y = -5$

- (D) $x + \frac{1}{2}y = 5$ (E) $-\frac{1}{2}x + y = 5$

If $y = mx + b$, m is the slope and b is the y-intercept. Substituting the given information gives $y = 2x + 5$. Look at the answers given to see which answer is equivalent to $y = 2x + 5$. The answer is C, $2x - y = -5$

12. Four numbers are written in a row. The average of the first two is 7. The average of the middle two is 2.3. The average of the last two is 8.4. What is the average of the first number and the last number ?

- (A) 13.1 (B) 8.9 (C) 3.7 (D) 7.7 (E) 17.7

Think of the four numbers as being a, b, c, and d.

If the average of the first two is 7, then $a + b = 14$. If the average of the middle two is 2.3, then $b + c = 4.6$. If the average of the last two is 8.4, then $c + d = 16.8$.

$$a + b = 14$$

$$b + c = 4.6$$

$$c + d = 16.8$$

Subtracting the middle equation from the top equation gives $a - c = 9.4$. If you then add the last equation to this result you get $a + d = 9.4 + 16.8$.

To find the average of the first and last numbers, you need to find $\frac{a+d}{2}$

which is equal to $\frac{9.4+16.8}{2} = 13.1$

13. If $2x^2 - 4 = 3x$ and r_1 and r_2 are the roots, then $r_1 + r_2$ equals _____.

(A) $\frac{3\sqrt{3}}{4}$ (B) $-\frac{1}{2}$ (C) 2 (D) -2 (E) $1\frac{1}{2}$

If $ax^2 + bx + c = 0$, the sum of the roots is $-\frac{b}{a}$.

$2x^2 - 4 = 3x$; $2x^2 - 3x - 4 = 0$ and the sum of the roots is
 $-\left(\frac{-3}{2}\right) = \frac{3}{2} = 1\frac{1}{2}$

14. A freight train left Harlingen at 5 a.m. traveling 30 mph. At 7 a.m. a passenger train traveling at 50 mph left the same station. What is the distance from the station when the passenger train overtakes the freight train ?

(A) 88 miles (B) 100 miles (C) 150 miles

(D) 180 miles (E) 240 miles

Let t = time it will take the passenger train to overtake the freight train.
Since Distance = Rate x Time ($D = RT$), then $30(t + 2) = D$ and $50t = D$.
 $30(t + 2) = 50t$; $30t + 60 = 50t$; $60 = 20t$; $t = 3$

Since it will take the passenger train 3 hours to overtake the freight train and the distanced traveled by the passenger train is $50t$, then $50(3) = 150$ miles is the desired distance.

15. One of the factors of $12xy - 9x + 8y - 6$ is :

(A) $4y - 3$ (B) $3x - 2$ (C) $3x - 3$

(D) $2y + 3$ (E) $6x - 2$

$12xy - 9x + 8y - 6$

$3x(4y - 3) + 2(4y - 3)$

$(3x + 2)(4y - 3)$; $4y - 3$ is one of the factors.

GEOMETRY

1. The point of concurrency of the perpendicular bisectors of the sides of a triangle is the _____.

(A) circumcenter (B) centroid (C) origin
(D) orthocenter (E) incenter

circumcenter : the point of concurrency of the perpendicular bisectors of the sides of a triangle

centroid : the point of intersection of the medians (sometimes called the median point)

orthocenter : the point of intersection of the altitudes of the triangle

incenter : the center of the circle inscribed in a triangle. The point where the angle bisectors of a triangle bisect each other

2. Gertrude Gardener built a rectangular goldfish pond that is 3' long, 2' wide, and 6" deep. She put 16 gallons of water in the pond. How far is the water from the top of the pond?

(A) $1\frac{1}{4}$ " (B) $1\frac{1}{2}$ " (C) $1\frac{3}{4}$ " (D) 2" (E) $2\frac{1}{4}$ "

Note : 1 gallon = 231 cubic inches. If 16 gallons are poured into the 3' long by 2' wide pond, the height, h can be found by

$$(3 \times 12)(2 \times 12)(h) = 16(231) ; 864h = 3696 ; h = 4.2778$$

The distance the water is from the top is $6 - 4.2778 = 1.7222$ which is approximately $1\frac{3}{4}$.

3. Father Time looked at his circular clock and saw the time to be 2:30. What was the measure of the angle between the big hand and the little hand at that time?

(A) 120° (B) 108° (C) 105° (D) 102° (E) 96°

At 2:30 the hour hand is halfway between 2 and 3. Since the measure between

hours on a clock is 30° , the hour hand is 15° from 3 o'clock. From 3 o'clock to 6 o'clock is 90° . Thus the measure of the angle of the big hand and the little hand of the clock is $15^\circ + 90^\circ$ or 105° .

4. A circle has a radius of 4 inches. If the diameter of the circle is increased by 25%, then the circumference of the larger circle is what percent of the circumference of the original circle?

(A) 112.5% (B) 125% (C) 137.5% (D) 150% (E) 162.5%

Note : The original diameter is d and the original circumference = $d\pi$. If the diameter is increased by 25%, it is the same as multiplying the diameter by 1.25. Thus the circumference of the larger circle will be $1.25d\pi$. The ratio of the circumference of the larger circle to the original circle is $\frac{1.25d\pi}{d\pi}$ which is equal to 1.25 or 125%.

5. A scalene triangle has side lengths of 4", 7", and 9". Find the area (nearest tenth).

(A) 10.0 in^2 (B) 18.0 in^2 (C) 31.5 in^2

(D) 13.4 in^2 (E) 29.0 in^2

To find the area of a triangle given sides a , b , and c use Heron's formula

(Area = $\sqrt{s(s-a)(s-b)(s-c)}$, where s = semi-perimeter = $\frac{a+b+c}{2}$).

$$S = \frac{4+7+9}{2} = 10 ; \text{ Area} = \sqrt{10(10-4)9(10-7)(10-9)} = 13.4$$

6. Find the midpoint of a segment with endpoints $(-3, -2)$ and $(7, -7)$.

(A) $(2, -4.5)$ (B) $(5, -4.5)$ (C) $(2, 4.5)$ (D) $(5, 2)$ (E) $(4.5, -5)$

If the endpoints of a segment are (x_1, y_1) and (x_2, y_2) the coordinates of the midpoint of the segment can be found by using the formula : $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

$$(\frac{-3+7}{2}, \frac{-2+-7}{2}) = (\frac{4}{2}, \frac{-9}{2}) = (2, -4.5)$$

7. If the height of a right cylindrical container is cut in half and the diameter is doubled, then what is the ratio of the volume of the original container to the volume of the new container?

(A) 1:1 (B) 2:1 (C) 1:4 (D) 4:1 (E) 1:2

Note : Volume of a right circular cylinder = $\pi r^2 h$

$$\frac{\text{volume(original)}}{\text{volume(new)}} = \frac{\pi r^2 h}{\pi(2r)^2(\frac{1}{2}h)} = \frac{\pi r^2 h}{2\pi r^2 h} = \frac{1}{2}, \text{ thus the ratio is 1:2.}$$

8. The sum of the interior angles of a regular convex polygon is 540 degrees. The polygon is a _____.

(A) Square (B) Pentagon (C) Hexagon (D) Octagon (E) Nonagon

The sum of the measures of the interior angles of a regular convex polygon with n sides is equal to $180(n - 2)$.

$$180(n - 2) = 540 ; \frac{180(n-2)}{180} = \frac{540}{180} ; n - 2 = 3 ; n = 5, \text{ Thus the polygon is a pentagon.}$$

9. The sides of a right triangle are m , n , and p where $m^2 + n^2 = p^2$. Find the length of the altitude to the hypotenuse.

(A) $\frac{mxn}{p}$ (B) $\frac{\sqrt{mn}}{p}$ (C) $\frac{p}{mxn}$ (D) $\frac{m+n}{p}$ (E) $\frac{p\sqrt{2}}{2mn}$

Note : The product of the altitude drawn to the hypotenuse and the hypotenuse is equal to the product of the legs.

h = length of the altitude drawn to the hypotenuse of a right triangle

$$hp = mn \quad \therefore h = \frac{mn}{p}$$

10. $\angle RST$ is inscribed in a circle. If $m\angle RST = 68^\circ$, then the measure of the intercepted arc is _____.

(A) 34° (B) 68° (C) 102° (D) 136° (E) 272°

If an angle is inscribed in a circle its measure is equal to one-half of the intercepted arc. If the measure of the inscribed angle is 68° , then the measure of the intercepted arc is $2(68^\circ)$ or 136° .

11. ΔABC exists such that $AB = BC = AC$. If the perpendicular bisector of $\angle A$ has a length of $3\sqrt{3}$ units, then the area of ΔABC is _____ square units.
- (A) $\frac{27\sqrt{3}}{4}$ (B) 3 (C) $\frac{\sqrt{3}}{2}$ (D) $9\sqrt{3}$ (E) $\frac{3}{4}$

Since all of the sides of the triangle are equal in length, the triangle is equilateral. the angle bisector of any of the angles is also a perpendicular bisector of a side and a height of the triangle. Given the height of an equilateral triangle the area is equal to $\frac{h^2\sqrt{3}}{3}$.

$$\text{Area} = \frac{(3\sqrt{3})^2\sqrt{3}}{3} = \frac{27\sqrt{3}}{3} = 9\sqrt{3}$$

12. A circle with a 7 inch diameter is cut out. A 2 inch border around the outer part of the circle is painted blue. A circular picture fills the unpainted part of the circle. What part of the circle is painted blue? (nearest whole percent)
- (A) 82% (B) 54% (C) 46% (D) 18% (E) 51%

$$\text{Blue area} = 3.5^2 \pi - 1.5^2 \pi$$

$$\text{Area of circle} = 3.5^2 \pi$$

$$\frac{3.5^2 \pi - 1.5^2 \pi}{3.5^2 \pi} \times 100 = 82\%$$

13. A 2" pipe is 100 feet long. How many quarts of water can it hold? (nearest quart)
- (A) 261 (B) 54 (C) 22 (D) 131 (E) 65

Note : 1 gallon = 231 cubic inches ; 1 gallon = 4 quarts ; 1 foot = 12 in.
Volume of a right circular cylinder = $\pi r^2 h$

$$\text{Volume} = \pi(2)^2(12 \times 100)$$

$$\text{Number of quarts the pipe can hold} = \frac{\pi(2)^2(12 \times 100)}{231} \times 4 = 261$$

14. One of two supplementary angles is 15° more than half of the other. Find the measure of the smaller angle's complement.

(A) 140° (B) 20° (C) 50° (D) 40° (E) 110°

supplementary angles : two angles the sum of whose measures is 180°

complementary angles : two angles the sum of whose measures is 90°

Let x = measure of one of the angles

$$15 + \frac{1}{2}x = \text{measure of the other angle}$$

$$x + (15 + \frac{1}{2}x) = 180 ; \frac{3}{2}x + 15 = 180 ; \frac{3}{2}x = 165 ;$$

$x = \frac{2}{3}(165) = 110$; Therefore the two angles have measure of 110° and 70° , respectively. The complement of the 70° angle is $90^\circ - 70^\circ$ or 20° .

15. A circle is inscribed in an equilateral triangle with side lengths of 6 units. Find the diameter of the circle.

(A) $\frac{\sqrt{3}}{2}$ (B) $2\sqrt{3}$ (C) $\sqrt{6}$ (D) $3\sqrt{2}$ (E) $\frac{\sqrt{6}}{3}$

Sketch a circle inscribed in an equilateral triangle. Keep in mind that the sides of the triangle are tangent to the circle, which means that they are perpendicular to a radius drawn to the tangent. This radius also bisects the side of the equilateral triangle. Draw a segment from the center to a vertex of the triangle and a radius to one of the sides. This will form a 30° - 60° - 90° triangle. The long leg will be 3 units long. The short leg is the radius.

$$\text{long leg} = \text{short leg} \times \sqrt{3}$$

$$3 = \text{radius} \times \sqrt{3}$$

$$\text{radius} = \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\text{diameter} = 2\sqrt{3}$$