

# NUMBER SENSE MAGIC

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### Multiplying two numbers close to 100 (Both numbers are less than 100)

Step # 1 : Find the difference of each number and 100. Multiply the result. This product will give the first two digits of the answer (the tens and units digits). If the product is greater than one hundred, carry the hundreds digit to Step #2.

Step # 2 : Subtract the difference of one of the numbers and one hundred from the other number. Add any carryover from Step #1. This result will be the remaining digits of the answer.

Example A :  $95 \times 98 = \underline{\hspace{2cm}}$

$$\text{Step \#1 : } (100 - 95)(100 - 98) = 5(2) = 10$$

$$\text{Step \#2 : } 95 - (100 - 98) = 95 - 2 = 93$$

Answer : 9310

Example B :  $89 \times 97 = \underline{\hspace{2cm}}$

$$\text{Step \#1 : } (100 - 89)(100 - 97) = 11(3) = 33$$

$$\text{Step \#2 : } 89 - (100 - 97) = 89 - 3 = 86$$

Answer : 8633

Example C :  $96 \times 95 = \underline{\hspace{2cm}}$

$$\text{Step \#1 : } (100 - 96)(100 - 95) = 4(5) = 20$$

$$\text{Step \#2 : } 96 - (100 - 95) = 96 - 5 = 91$$

Answer : 9120

Example D :  $87 \times 88 = \underline{\hspace{2cm}}$

$$\text{Step \#1 : } (100 - 87)(100 - 88) = 13(12) = 156 \text{ (Write Down the 56 and carryover the 1 to Step \#2.)}$$

$$\text{Step \#2 : } 87 - (100 - 88) + 1 \text{ (carryover)} = 87 - 12 + 1 =$$

Answer : 7656

## Subtracting whole numbers

In teaching students a method for subtracting faster, I recommend that they first practice subtracting two digits at a time. The first examples shown should not require borrowing from the hundreds place.

Example A :  $945 - 532 = \underline{\hspace{2cm}}$

Step #1 :  $45 - 32 = 13$

Step #2 :  $9 - 5 = 4$

Answer : 413

Example B :  $859 - 246 = \underline{\hspace{2cm}}$

Step #1 :  $59 - 46 = 13$

Step #2 :  $8 - 2 = 6$

Answer : 613

Example C :  $936 - 512 = \underline{\hspace{2cm}}$

Step #1 :  $36 - 12 = 24$

Step #2 :  $9 - 5 = 4$

Answer : 424

Example D :  $789 - 152 = \underline{\hspace{2cm}}$

Step #1 :  $89 - 52 = 37$

Step #2 :  $7 - 1 = 6$

Answer : 637

Example E :  $5736 - 5221 = \underline{\hspace{2cm}}$

Step #1 :  $36 - 21 = 15$

Step #2 :  $57 - 52 = 5$

Answer : 515

Example D :  $6729 - 4318 = \underline{\hspace{2cm}}$

$$\begin{aligned}\text{Step \#1} &: 29 - 18 = 11 \\ \text{Step \#2} &: 67 - 43 = 24\end{aligned}$$

Answer : 2411

Now students need to learn how to handle situations where borrowing from the hundreds place is required.

$$\text{Example A : } 7413 - 5988 = \underline{\hspace{2cm}}$$

Step #1: Try to find the difference of the first two digits of each number (the tens place and units place), If the number being subtracted is larger than the other number, find the difference of this number and one hundred and add this to the other number.

$$13 + (100 - 88) = 13 + 12 = 25$$

Step #2 : Find the difference of the next two digits less 1.

$$(74 - 59) - 1 = 15 - 1 = 14$$

Step #3 : Repeat Step #1 and Step #2 if necessary

Answer : 1425

$$\text{Example B : } 967 - 295 = \underline{\hspace{2cm}}$$

$$\text{Step \#1 : } 67 + (100 - 95) = 67 + 5 = 72$$

$$\text{Step \#2 : } (9 - 2) - 1 = 7 - 1 = 6$$

Answer : 672

$$\text{Example C : } 805 - 279 = \underline{\hspace{2cm}}$$

$$\text{Step \#1 : } 05 + (100 - 79) = 05 + 21 = 26$$

$$\text{Step \#2 : } (8 - 2) - 1 = 6 - 1 = 5$$

Answer : 526

$$\text{Example D : } 8756 - 7588 = \underline{\hspace{2cm}}$$

$$\text{Step \#1 : } 56 + (100 - 88) = 56 + 12 = 68$$

$$\text{Step \#2 : } (87 - 75) - 1 = 12 - 1 = 11$$

Answer : 1168

Example E :  $7623 - 5991 = \underline{\hspace{2cm}}$

Step #1 :  $23 + (100 - 91) = 23 + 9 = 32$

Step #2 :  $(76 - 59) - 1 = 17 - 1 = 16$

Answer : 1632

Example F :  $6537 - 3265 = \underline{\hspace{2cm}}$

Step #1 :  $37 + (100 - 65) = 37 + 35 = 72$

Step #2 :  $(65 - 32) - 1 = 33 - 1 = 32$

Answer : 3272

Example G :  $3187 - 2695 = \underline{\hspace{2cm}}$

Step #1 :  $87 + (100 - 95) = 87 + 5 = 92$

Step #2 :  $(31 - 26) - 1 = 5 - 1 = 4$

Answer : 492

Example H :  $9623 - 7583 = \underline{\hspace{2cm}}$

Step #1 :  $23 + (100 - 83) = 23 + 17 = 40$

Step #2 :  $(96 - 75) - 1 = 21 - 1 = 20$

Answer : 2040

Example I :  $5638 - 2369 = \underline{\hspace{2cm}}$

Step #1 :  $38 + (100 - 69) = 38 + 31 = 69$

Step #2 :  $(56 - 23) - 1 = 33 - 1 = 32$

Answer : 3269

Example J :  $7859 - 6785 = \underline{\hspace{2cm}}$

Step #1 :  $59 + (100 - 85) = 59 + 15 = 74$

Step #2 :  $(78 - 67) - 1 = 11 - 1 = 10$

Answer : 1074

## CHANGING FROM BASE 10 TO ANOTHER BASE

Example A : 45 base 10 = \_\_\_\_\_ base 6.

Step #1 : The first digit of the answer (from right to left) is equal to the remainder when you divide the given number by the base.

$$45 \div 6 = 7, \text{ remainder } 3. \text{ Write down the } 3.$$

Step #2 : Divide the quotient in Step #1 by the base. The remainder is the next digit of the answer.

$$7 \div 6 = 1, \text{ remainder } 1. \text{ Write down the } 1.$$

Step #3 : Continue dividing the quotient of the previous step by the base, always writing down the remainder until the final digit of the answer is found.

$$1 \div 6 = 0, \text{ remainder } 1. \text{ Write down the } 1.$$

Answer : 113

Example B : 73 base 10 = \_\_\_\_\_ base 9.

Step #1 : The first digit of the answer (from right to left) is equal to the remainder when you divide the given number by the base.

$$73 \div 9 = 8, \text{ remainder } 1. \text{ Write down the } 1.$$

Step #2 : Divide the quotient in Step #1 by the base. The remainder is the next digit of the answer.

$$8 \div 9 = 0, \text{ remainder } 8. \text{ Write down the } 8.$$

Answer : 81

Example C : 38 base 10 = \_\_\_\_\_ base 4.

Step #1 : The first digit of the answer (from right to left) is equal to the remainder when you divide the given number by the base.

$38 \div 4 = 9$ , remainder 2. Write down the 2.

Step #2 : Divide the quotient in Step #1 by the base. The remainder Is the next digit of the answer.

$9 \div 4 = 2$ , remainder 1. Write down the 1.

Step #3 : Continue dividing the quotient of the previous step by the base, always writing down the remainder until the final digit of the answer is found.

$2 \div 4 = 0$ , remainder 2. Write down the 2.

Answer : 212

Example D : 69 base 10 = \_\_\_\_\_ base 6.

Step #1 : The first digit of the answer (from right to left) is equal to the remainder when you divide the given number by the base.

$69 \div 6 = 11$ , remainder 3. Write down the 3.

Step #2 : Divide the quotient in Step #1 by the base. The remainder Is the next digit of the answer.

$11 \div 6 = 1$ , remainder 5. Write down the 5.

Step #3 : Continue dividing the quotient of the previous step by the base, always writing down the remainder until the final digit of the answer is found.

$1 \div 6 = 0$ , remainder 1. Write down the 1.

Answer : 153

## Special problems

1.  $1 + 2 + 3 + 4 + \dots + n = \underline{\hspace{2cm}}$ ,

Rule :  $\frac{n(n+1)}{2}$

Example A :  $1 + 2 + 3 + \dots + 10 = \underline{\hspace{2cm}}$ .

Solution :  $\frac{10(10+1)}{2} = \frac{110}{2} = 55$

Example B :  $1 + 2 + 3 + \dots + 19 = \underline{\hspace{2cm}}$ .

Solution :  $\frac{19(19+1)}{2} = \frac{380}{2} = 190$

Example C :  $1 + 2 + 3 + \dots + 15 = \underline{\hspace{2cm}}$ .

Solution :  $\frac{15(15+1)}{2} = \frac{240}{2} = 120$

2.  $2 + 4 + 6 + \dots + n = \underline{\hspace{2cm}}$ .

Rule :  $\frac{n(n+2)}{4}$

Example A :  $2 + 4 + 6 + \dots + 20 = \underline{\hspace{2cm}}$ .

Solution :  $\frac{20(20+2)}{4} = \frac{20}{2} \left(\frac{22}{2}\right) = 10(11) = 110$

Example B :  $2 + 4 + 6 + \dots + 14 = \underline{\hspace{2cm}}$ .

Solution :  $\frac{14(14+2)}{4} = \frac{14}{2} \left(\frac{16}{2}\right) = 7(8) = 56$

Example C :  $2 + 4 + 6 + \dots + 24 = \underline{\hspace{2cm}}$ .

Solution :  $\frac{24(24+2)}{4} = \frac{24}{2} \left(\frac{26}{2}\right) = 12(13) = 156$

6.  $421 \div 9 = \underline{\hspace{2cm}}$  (mixed number)

Step #1 : Find the sum of the digits of the given number. This will be the numerator of a fraction whose denominator is 9. If the result is a proper fraction write this as part of the answer. If the result is an improper fraction, convert it into a mixed number ; write down the fractional part, then carry the whole number part to Step #2.

$$\frac{4 + 2 + 1}{9} = \frac{7}{9}$$

Step #2 : Find the sum of the hundreds digit and the tens digit (plus any carryover from Step #1). This is the units digit of the answer.

$$(4 + 2) + 0 = 6$$

Step #3 : The hundreds digit of the number being divided by 9 (plus any carryover from Step #2) will be the tens digit of the answer.

$$4 + 0 = 4$$

$$\text{Answer : } 46\frac{7}{9}$$

Example A :  $435 \div 9 = \underline{\hspace{2cm}}$  (mixed number)

Step #1 :  $\frac{4 + 3 + 5}{9} = \frac{12}{9} = \frac{4}{3} = 1\frac{1}{3}$  ; Write down the  $\frac{1}{3}$  and carryover the 1.

Step #2 :  $4 + 3 + 1(\text{carryover}) = 8$

Step #3 :  $4 + 0(\text{no carryover}) = 4$

$$\text{Answer : } 48\frac{1}{3}$$

Example B :  $523 \div 9 = \underline{\hspace{2cm}}$  (mixed number)

Step #1 :  $\frac{5 + 2 + 3}{9} = \frac{10}{9} = 1\frac{1}{9}$  ; Write down the  $\frac{1}{9}$

and carryover the 1.

$$\text{Step \#2 : } 5 + 2 + 1(\text{carryover}) = 8$$

$$\text{Step \#3 : } 5 + 0(\text{no carryover}) = 5$$

$$\text{Answer : } 58 \frac{1}{9}$$

Example C :  $426 \div 9 = \underline{\hspace{2cm}}$  (mixed number)

$$\text{Step \#1 : } \frac{4+2+6}{9} = \frac{12}{9} = 1 \frac{3}{9} = 1 \frac{1}{3}; \text{ Write down the } \frac{1}{3} \text{ and carryover the 1.}$$

$$\text{Step \#2 : } 4 + 2 + 1(\text{carryover}) = 7$$

$$\text{Step \#3 : } 4 + 0(\text{no carryover})$$

$$\text{Answer : } 47 \frac{1}{3}$$

Example D :  $314 \div 9 = \underline{\hspace{2cm}}$  (mixed number)

$$\text{Step \#1 : } \frac{3+1+4}{9} = \frac{8}{9}$$

$$\text{Step \#2 : } 3 + 1 + 0(\text{no carryover}) = 4$$

$$\text{Step \#3 : } 3 + 0(\text{no carryover}) = 3$$

$$\text{Answer : } 34 \frac{8}{9}$$