MATHEMATICS TIPS (AUGUST 2017)

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1. The angles of a triangle are in a ratio of 1:2:3. What is the supplement to the smallest of these angles?

   (A) 90° (B) 120° (C) 150° (D) 60° (E) 180°

   $1x + 2x + 3x = 180$; $6x = 180$; $x = 30$

   The angles of the triangle are 30°, 60°, and 90°.

   Since supplementary angles add up to 180, then the supplement of the smallest angle is $180 - 30 = 150$.

2. Melissa went to the zoo and saw a cage that had birds flying in the air as well as lizards crawling around on the ground. Melissa counted 40 heads and 112 legs. How many more birds did Melissa see than lizards?

   (A) 12 (B) 6 (C) 10 (D) 8 (E) 4

   $B + L = 40$

   $2B + 4L = 112$

Eliminate one of the variables. To eliminate the B's, multiply the top equation by – 2, then add the two equations.

$$-2B - 2L = -80$$
$$-2B + 4L = 112$$

$2L = 32$; $L = 16$; Since there are 16 lizards,then there are $40 - 16 = 24$ birds. Thus there are $24 - 16 = 8$ more birds than lizards.

3. It takes Bryce 30 minutes to mow his front yard. It takes his father 20 minutes to move their front yard. If they work together, how many minutes would it take both Bryce and his father to mow their front yard?

(A) 18 minutes (B) 16 minutes (C) 14 minutes

(D) 12 minutes (E) 10 minutes

$$\frac{Both}{Aalone} + \frac{Both}{Balone} = 1$$

$$\frac{x}{30} + \frac{x}{20} = 1$$

Eliminate the denominators by multiplying by the least common multiple of 30 and 20.

$$\text{LCM}(30, 20) = 60$$

$$60\left(\frac{x}{30}\right) + 60\left(\frac{x}{20}\right) = 60(1)$$

$2x + 3x = 60$; $5x = 60$; $x = 12$

4. If $x + y = 7$ and $xy = 10$, what is the value of $x^2 + y^2$?

(A) 14 (B) 49 (C) 29 (D) 289 (E) 39

$(x + y)^2 = 7^2$

$x^2 + 2xy + y^2 = 49$

$x^2 + 2(10) + y^2 = 49$

$x^2 + y^2 = 29$

5. There are eight players on the Mighty Tykes Soccer Team. If only five players can be on the field at once, how many ways can the five players be selected?

(A) 40 (B) 6,720 (C) 13 (D) 56 (E) 48

$${}_8C_5 = \frac{8!}{(8-5)!5!} = \frac{8!}{3!5!} = \frac{(8)(7)(6)}{(3)(2)(1)} = (8)(7) = 56$$

WHY IS DIVISION BY 0 UNDEFINED?

There are numerous instances in mathematics where common errors occur. In my opinion, the most frequent error that I ever saw in my career, occurred when a number was divided by 0. Division by 0 occurs in elementary mathematics, coordinate geometry (finding slope) and Calculus (finding limits). Explaining why division by 0 is undefined is something that students should be introduced to as soon as division of whole numbers is taught. In general, students learn that division is another form of multiplication. In the problem, $28 \div 4 = 7$, 28 is the dividend, 4 is the divisor and 7 is the quotient. Students learn that the product of the quotient and the divisor is equal to the dividend. Thus, $7 \times 4 = 28$. If I wanted students to discover why you can’t divided by 0, I would ask them a series of basic division problems. For example, in the problem $63 \div 9$, students need to find a number that when multiplied by 9 will equal 63. That number is 7, since $9 \times 7 = 63$. After a student confirms that the product of the quotient and the divisor is equal to the dividend, they should be asked the following problem. $7 \div 0$ is equal to what? Students will try to think of a number that will equal to 7 when multiplied by 0. Since the product of 0 and any number is 0, they will conclude that there is no number that can be multiplied by 0 that will equal 7. At this point, you inform them that division by zero is undefined. In other words, there is no way that you can divide by 0. The previous problem should be followed by the following question. $0 \div 7$ is equal to what? In this problem, you want to find a number that when multiplied by 7 is 0. The answer to this is 0.Thus, $0 \div 7 = 0$. The concept that division by 0 is undefined should be reinforced at every grade level in every math course. It is also something that I parents should teach their children.

WHY IS THE $\sqrt{x^2}$ NOT EQUAL TO X?

When teaching mathematics courses, starting with Algebra 1, students frequently say that $\sqrt{x^2}$ = x, which is not true. In fact, $\sqrt{x}$ = |x|. Students should learn, that if you are looking for an even root and the result is a variable with an odd exponent, the answer requires an absolute value sign. I would encourage you to illustrate why $\sqrt{x^2} \neq x$ by letting x = - 9. Notice that if $\sqrt{x^2} = x$, then $\sqrt{(-9)^2}$ would equal – 9. By definition, the square of a number is nonnegative. If you solve $\sqrt{(-9)^2}$ you will notice that it is equal to $\sqrt{81}$ which is equal to 9. Thus, $\sqrt{x^2} \neq x$. Now, lets demonstrate that $\sqrt{x^2} = |x|$ by letting x = - 9. If $\sqrt{x^2} = |x|$, then $\sqrt{(-9)^2} = |-9| = 9$. This can be confirmed, because $\sqrt{(-9)^2} = \sqrt{81} = 9$. Another way that you can show that $\sqrt{x^2} \neq x$ is by comparing the graphs of $y = \sqrt{x^2}$ and $y = x$ on a graphing calculator. You will observe that their graphs are different. On the other hand, you can confirm that $\sqrt{x^2} = |x|$ by comparing the graph of $y = \sqrt{x^2}$ and $y = |x|$ because you will notice that their graphs are the same. The concept that $\sqrt{x^2} = |x|$ should be reinforced in courses from Algebra 1 through Calculus.

COMMON ERROR WHEN SOLVING RADICAL EQUATIONS

Students that solve radical equations are confronted with a situation where a common error occurs. In general, to solve a radical equation you isolate the radical, square both sides of the equation, then solve for x. If this is done when solving the following problem, students usually make an error. Let's solve $2\sqrt{x} + 17 = 3$. Isolating the radical we have $2\sqrt{x} = 3 - 17$; $2\sqrt{x} = -14$; $\sqrt{x} = \frac{-14}{2}$; $\sqrt{x} = -7$. At this point, the student should realize that the square root of a number must be nonnegative, which means that $\sqrt{x} \neq -7$, thus the equation has no solution. In many cases, the students will square both sides of the equation $\sqrt{x} = -7$. $\left(\sqrt{x}\right)^2 = (-7)^2$; $x = 49$. When solving radical equations, sometimes you get an erroneous solution. That is why it is important that you check the answer. Let $x = 49$ in the equation $2\sqrt{x} + 17 = 3$ to determine if the answer is correct. $2\sqrt{49} + 17 = 2(7) + 17 = 14 + 17 = 31$. Since the equation is suppose to equal 3, not 31, that means that $x \neq 49$. Teachers and parents should alert their students or children about this type of error so they can avoid it.

COMMON ERROR WHEN SOLVING ABSOLUTE VALUE EQUATIONS

Students solving absolute value equations many times overlook a situation that results in a common error. Teachers and parents need to emphasize that the absolute value of a number is nonnegative (in other words, the absolute value of a number can't be negative). The following example will illustrate the common error. Focus on each step as you solve $2|x + 3| + 29 = 17$. $2|x + 3| = 17 - 39$; $2|x + 3| = -22$; $|x + 3| = \frac{-22}{2}$; $|x + 3| = -11$. At this point, the student has been focused on making sure that each step is being done correctly. Many students disregard the fact that $|x + 3| = -11$ shows that the absolute value of x) + 3 is a negative. But that is impossible. At this time, students should say that the problem has "no solution" or they can state that the answer is the empty set, { }. If students disregard the fact that the absolute value is nonnegative, the only other thing that I can recommend is that if values for x are obtained, that they be substituted into the original equation to determine if the solutions are valid. Obviously the answers would not be verified and the hope would be that the student would take a second look at where the error occurred.

COMMON ERRORS IN ALGEBRA

Other common errors that students constantly make in classes beginning with Algebra 1, involve $(a + b)^2$ and two properties the confusion between the commutative and the symmetric properties. Many students incorrectly think that $(a + b)^2$ is equal to $a^2 + b^2$. If students used the distributive property or what has become known as the FOIL method of multiplying binomials, they would discover that $(a + b)^2 = (a + b)(a + b) = (a)(a) + (a)(b) + (b)(a) + (b)(b) = a^2 + 2ab + b^2$. Students should memorize $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$. The other error occurs between a student's confusion between the commutative property and the symmetric property. In the commutative property of addition, $a + b = b + a$ and the commutative property of multiplication is $ab = ba$. On the other hand, the symmetric property states that if $a = b$, then $b = a$. The symmetric property is the property that allows for the flipping of an equation. For example, let's look at the solution of $2x + 21 = 5x$. $21 = 5x - 2x$; $21 = 3x$; $\frac{21}{3} = \frac{3x}{3}$; $7 = x$; $x = 7$. In general, when an equation is solved, the variable is traditionally written on the left side of the equation, The aforementioned errors should be emphasized by teachers and parents to their students or children.